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DESIGN OPTIMIZATION OF SANDWICH PANELS

Abstract

The paper deals with design optimization of sandwich structure made of laminate outer layers and PUR foam core. The thickness of outer layers with the known fibre orientation angle of individual laminae, referred to as the thickness variable, will be used as design variable. The optimization problem with displacement constraint will be formulated to minimize the weight of sandwich with laminate outer layers. The design is optimized using continuous design variable.

Keywords

Optimization, sandwich panel, maximum displacement criterion, thickness design variable, weight objective function.

1 INTRODUCTION

The optimization of a composite plate is important analysis for design of structures ranging from aircrafts to civil engineering structures.

The design optimization problem of current interest is the minimization of the weight function for a sandwich composite plate. This is a design optimization problem which optimizes the thickness of the sandwich layers to give the minimum weight. Of greater interest to current study are the works on the design optimization of composite sandwich plates where the thickness of outer layers and the core are taken as the design variables.

2 SANDWICH THEORY

The typical sandwich structure compounds of three layers. The outer layers are made of a material that has high strength (fiber reinforced laminates), which can transfer axial forces and bending moments, while the core is made of lightweight materials such as foam, alder wood etc. The material used in sandwich core must be resistant to compression and capable of transmitting shear. The thin cover sheets, i.e. the layers 1 and 3, have the thicknesses h_1 for the lower skin and h_3 for the upper skin. The thickness of the core is h_2 (Fig. 1). In a general case, h_1 does not have to be equal to h_3, but in the most important practical case of symmetric sandwiches $h_1 = h_3$.

Most sandwich structures can be modeled and analyzed using the shear deformation theory for laminate plates [1-3].

For the resultants **N** and **M** the integration is carried out over the sheets only and for the transverse shear force over the core. The constitutive equations for a sandwich are written in the hypermatrix form

$$\begin{pmatrix} \mathbf{N} \\ \mathbf{M} \\ \mathbf{V} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{0} \\ \mathbf{C} & \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}^s \end{pmatrix} \begin{pmatrix} \boldsymbol{\varepsilon}_0 \\ \boldsymbol{\kappa} \\ \boldsymbol{\gamma} \end{pmatrix} \tag{1}$$

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where **N**, **M**, **V** are the vectors of normal forces, bending moments and transverse shear forces, respectively and $\boldsymbol{\varepsilon}_0, \boldsymbol{\kappa}, \boldsymbol{\gamma}$ are the vectors of mid-plane strains, curvatures and transverse shear strains, respectively. The stiffness coefficients are calculated as

$$A_{ij} = A_{ij}^{(1)} + A_{ij}^{(3)}, \qquad B_{ij} = \frac{1}{2} h^{(2)} \left(A_{ij}^{(3)} - A_{ij}^{(1)} \right), \tag{2}$$

$$C_{ij} = C_{ij}^{(1)} + C_{ij}^{(3)}, \qquad D_{ij} = \frac{1}{2} h^{(2)} \left(C_{ij}^{(3)} - C_{ij}^{(1)} \right), \tag{3}$$

$$A_{ij}^{s} = E_{ij}^{s}\, h^{(2)}; \text{ i,j} = 4{,}5 \tag{4}$$

where E_{ij}^{s} is the transverse shear modulus of the core.

3 SIZING OPTIMIZATION

The optimization process is applied to the approximate problem represented by the polynomial approximation. The coefficients of the polynomial function are determined by the least squares regression.

For regression analysis the singular value decomposition is used. When the objective function and constraints are approximated and their gradients with respect to the design variables are calculated based on chosen approximation, it is possible to solve the approximate optimization problem.

One of the algorithms used in the optimization module is called the Modified Feasible Direction method (MFD). The solving process is iterated until convergence is achieved.

It is important to distinguish the iteration inside the approximate optimization from the loop in the overall optimization process. Convergence of MFD to the optimum is checked by criteria of maximum iterations and criteria of objective function changes.

Besides the previously mentioned criteria, the Kuhn-Tucker conditions necessary for optimality must be satisfied.

Convergence or termination checks are performed at the end of each optimization loop in general optimization. The optimization process continues until either convergence or termination occurs.

4 MODELING AND SOLUTION OF SANDWICH PLATE

For the numerical solution the simply supported panel with laminate facings was used [4, 5]. Panel length is L = 3750 mm, nominal width is B = 1000 mm. Thickness of the facings is $h_1 = h_3$ and core is h_2 (Fig. 1). On the panel affects uniform static wind load with intensity of 2 kPa in the bending plane. The laminate Carbon/epoxy facings are composed of eight identical thickness layers of a symmetric laminate $[0/\pm45/90]_s$.

It was considered the carbon fibres in epoxy matrix, while unidirectional laminate layer has characteristics:

E_f = 230 GPa; E_m = 3 GPa; ν_f = 0.2; ν_m = 0.3; V_f = 0.6; ρ_k = 1580 kg/m^3.

Sandwich core, consisting of polystyrene, has material constants: E_P = 16 MPa;

ν_P = 0.3; ρ_P = 150 kg/m^3.

Laminate properties were determined by homogenization techniques [4, 12]. Computational program MATLAB was used to calculate the effective material properties of laminate facings. Numerical solutions were conducted through the COSMOS/M program. STAR module for solving linear static was used for calculations. There were used finite elements of the type SHELL4L. These are the 4-node multi-layer quadrilateral elements with membrane and bending response and can be enter up to fifty layers.

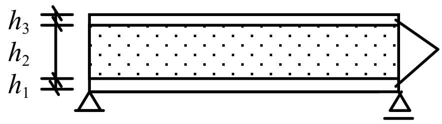

Fig. 1: Scheme of sandwich structure

Design optimization problems can be written as follows:

Optimization problem 1:

$$F(\boldsymbol{X}) = G(h_1) \to \min \quad [\mathrm{N}]$$
$$1 \cdot 10^{-4} \le h_1 \le 0.01 \, [\mathrm{m}]$$
$$h_2 = 0.1 \; [\mathrm{m}]$$
$$0 \le w \le 0.0375 \quad [\mathrm{m}]$$

Optimization problem 2:

$$F(\boldsymbol{X}) = G(h_2) \to \min \quad [\mathrm{N}]$$
$$1 \cdot 10^{-2} \le h_2 \le 0.2 \, [\mathrm{m}]$$
$$h_1 = 0.001 \; [\mathrm{m}]$$
$$0 \le w \le 0.0375 \quad [\mathrm{m}]$$

Optimization problem 3:

$$F(\boldsymbol{X}) = G(h_1, h_2) \to \min \quad [\mathrm{N}]$$
$$5 \cdot 10^{-4} \le h_1 \le 0.002 \; [\mathrm{m}]$$
$$5 \cdot 10^{-2} \le h_2 \le 0.2 \quad [\mathrm{m}]$$
$$0 \le w \le 0.0375 \quad [\mathrm{m}]$$

The initial values and bounds of design variables, constraints and the objective function are shown in the Table 1 for optimization problem 1.

Tab. 1: Summary of results of the optimization problem 1

Optimization parameters		Initial values	Final values	Tolerance τ
Design variable	h_1 [m]	0.001	$5.683 \cdot 10^{-4}$	$1 \cdot 10^{-5}$
Objective function	G [N]	573.75	568.983	$1 \cdot 10^{-3}$
Constraint	w [m]	0.02378	0.0375	$3.75 \cdot 10^{-4}$

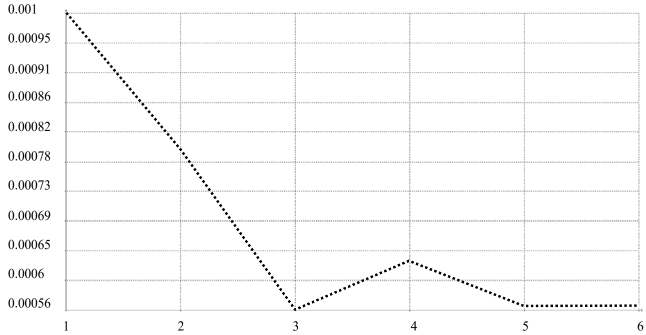

Fig. 2: Variation of design variable h_1 [m] during the optimization process 1

The initial values and bounds of design variables, constraints and the objective function are shown in the Table 2 for optimization problem 2.

Tab. 2: Summary of results of the optimization problem 2

Optimization parameters		Initial values	Final values	Tolerance τ
Design variable	h_2 [m]	0.1	$7.755 \cdot 10^{-2}$	$1 \cdot 10^{-5}$
Objective function	G [N]	573.75	447.445	$1 \cdot 10^{-3}$
Constraint	w [m]	0.02378	0.0375	$3.75 \cdot 10^{-4}$

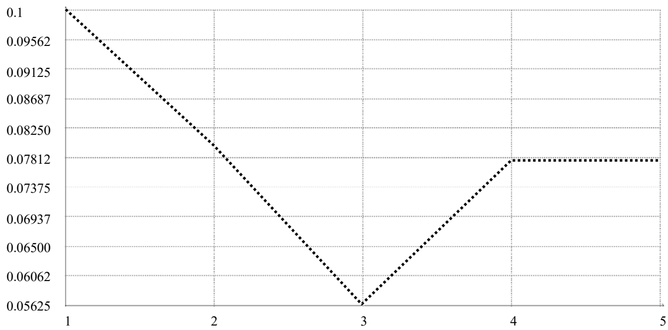

Fig. 3: Variation of design variable h_2 [m] during the optimization process 2

The initial values and bounds of design variables, constraints and the objective function are shown in the Table 3 for optimization problem 3.

Tab. 3: Summary of results of the optimization problem 3

Optimization parameters		Initial values	Final values	Tolerance τ
Design variable	h_1 [m]	0.001	0.002	$1 \cdot 10^{-5}$
Design variable	h_2 [m]	0.158	0.05067	$1 \cdot 10^{-5}$
Objective function	G [N]	900	341.31	$1 \cdot 10^{-3}$
Constraint	w [m]	0.01092	0.0375	$3.75 \cdot 10^{-4}$

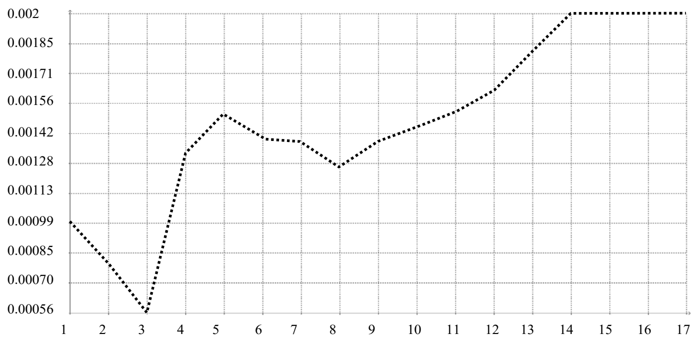

Fig 4: Variation of design variables h_1 [m] during optimization process 3

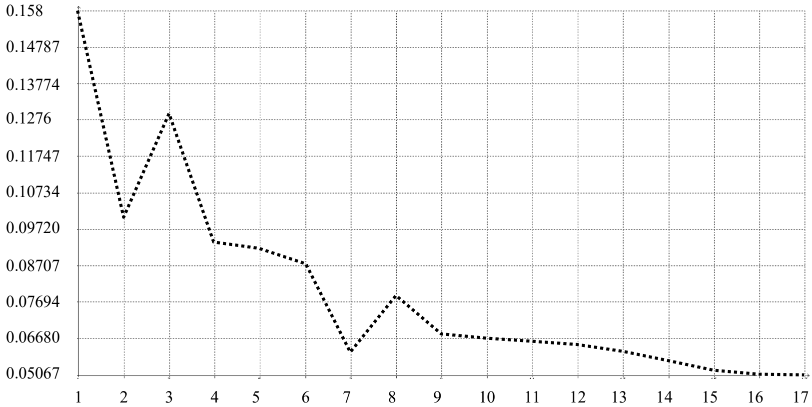

Fig. 5: Variation of design variables h_2 [m] during optimization process 3

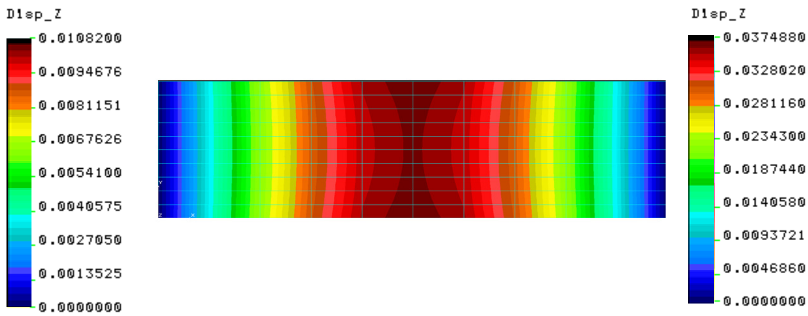

Fig. 6: Contour plot of deflections w before and after the optimization process 3

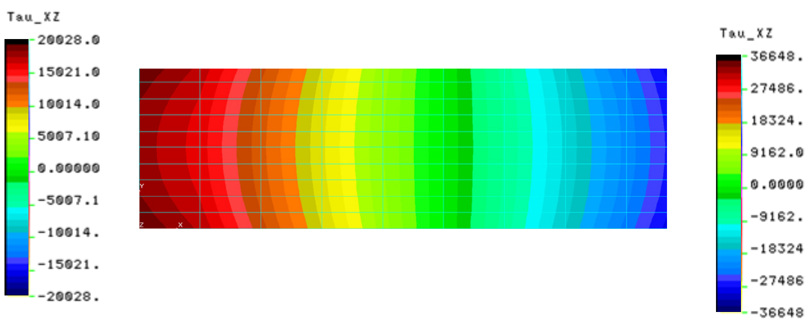

Fig. 7: Contour plot of stresses τ_{xz} at the bottom of core layer before and after the optimization process 3

6 CONCLUSIONS

The first order shear laminate theory was used by the FEM analysis of the problem [5-11]. The problem was formulated as a minimum weight of simply supported rectangular sandwich plate subject to deflection constraint in the middle of the plate. Design variable were thicknesses h_1 and h_2 of sandwich layers. The optimal problem was solved using SLP and MFD method with maximum 70 iterations in each own optimization loop. In the Figs. 2-5 there are depicted variations of design variables during the optimization processes 1, 2 and 3, respectively. Initial and final values of optimization processes 1, 2 and 3 are shown in the Tables 1, 2 and 3, respectively. Contour plot of deflections w and stresses τ_{xz} at the bottom of the core layer before and after optimization process 3 are illustrated in the Figs. 6 and 7, respectively. The most general optimization procedure is optimization process 3 with two design variables, where designer can optimize whole thickness of sandwich panel within to take into account both constraints for thicknesses h_1 and h_2. There was not taken into account a hygrothermal effect of environment. Only static analysis under mechanical loading was performed.

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